– Formation and evolution of galaxies –
Exam 2024/2025

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31^{st} of October 2024, 15:00-17:00

Important notes

– No internet-connected devices are allowed.
– This is a closed-book exam; only one double-sided A4 sheet of notes is permitted.
– Write your name and student number on *all* the pages that you hand in.
– Please make sure that your handwriting is easy to read!

This exam consists of 6 very concise questions and 2 exercises.
Each question is worth a maximum of 0.5 points. Exercises 1 and 2 are worth a maximum of 3 and 4 (1 per item) points, respectively. The final grade of the exam is the sum of all the points.

Tips:
– Give very concise answers to the questions. Focus only on precisely what is asked. If not requested, do not embark on calculations or derivations.
– Read the text of the questions and exercises very carefully.
– If, as a result of a calculation, you obtain values that do not make sense to you but you cannot find the error, write a comment indicating the approximate value that you would have expected and why.

Questions

Q1

Galaxy morphology tends to change when moving from the field to dense environments such as galaxy clusters; this is the essence of the morphology-density relation. This morphological variation also corresponds to changes in the properties of (i) the stellar populations of galaxies and (ii) their gas content. Describe how (i) and (ii) change between the field and galaxy clusters, and mention a possible physical cause.

Q2

A damped Lyman-α (DLA) system can be easily identified in the spectrum of a QSO as a broad absorption feature at observed wavelength $\lambda_{\rm obs} = \lambda_{\rm Ly\alpha}(1+z_{\rm abs})$, where $\lambda_{\rm Ly\alpha} = 1216\,\text{Å}$ is the rest-frame wavelength of the Lyman-α transition and $z_{\rm abs}$ is the redshift of the absorbing feature. Do you expect such a feature to more visible in the spectrum of a QSO at $z = 2.5$ or $z = 6$? Motivate your answer.

Q3

The power spectrum that describes the evolution of dark matter fluctuations, $P(k,t)$, is obtained by multiplying the primordial power spectrum $P_{\rm prim}$ by the square of two functions: the growth factor D and the transfer function T. What are the dependencies of these functions, and what is the role of the transfer function?

Q4

Consider the phenomenon of universal re-ionisation, when the intergalactic medium changes from being largely neutral to largely ionised. This transition is thought to be driven by the first galaxies formed around $z \sim 10$. Which physical effect between shocks and radiation was more important for re-ionisation to occur and why does it occur in patches?

Q5

In chemical evolution models, the evolution of the abundance of a certain element ($Z_{\rm i} M_{\rm g}$, where $M_{\rm g}$ is the gas mass and $Z_{\rm i}$ is the fractional abundance of the element i) is influenced by gas inflow and outflow. The inflow and outflow rates are written as $Z_{\rm i,in} \dot{M}_{\rm in}$ and $Z_{\rm i,out} \dot{M}_{\rm out}$, respectively. In order simplify the problem, which of the following equalities would be reasonable to employ: $Z_{\rm i,in} = Z_{\rm i}$, $Z_{\rm i,out} = Z_{\rm i}$ or $Z_{\rm i,in} = Z_{\rm i,out}$? Motivate your answer.

Q6

The stellar component of galaxies tends to grow inside-out. Give a possible physical reason for this phenomenon and explain whether it would be more effective for star-forming or early-type galaxies.

Exercises

E1

Using ALMA observations of the [CII] fine-structure emission line, the rotation curve of a disc galaxy at $z = 4$ has been derived. The gas in this galaxy is observed to rotate at a speed $v_{\rm rot} = 300\,{\rm km\,s^{-1}}$ that remains constant with radius (flat rotation curve) out to a (physical) distance of $R_{\rm out} = 5\,{\rm kpc}$ from the centre of the galaxy. Here, $R_{\rm out}$ represents the extent of the gaseous disc. The gas emitting in [CII] is cold and in nearly circular orbits; thus, its rotation can be considered a good measure of the circular speed, i.e. $v_{\rm rot} = v_{\rm c}$.

(i) Using independent observations at different wavelengths around the optical/NIR rest-frame and employing the SED fitting technique, the stellar mass of the galaxy is estimated to be $M_\star = 4.5 \times 10^{10}\,M_\odot$. The gas mass is, instead, estimated to be $M_{\rm g} = 3 \times 10^{10}\,M_\odot$. Assuming spherical symmetry, use the measured $v_{\rm rot}$ to calculate the total (dynamical) mass, the dark matter mass, and the baryon fraction within $R_{\rm out}$, $f_{\rm b,R_{out}}$.

(ii) Next, we estimate the virial mass $M_{\rm vir}$, i.e. the dark matter mass out to the virial radius $r_{\rm vir}$, and the baryon fraction within $r_{\rm vir}$, $f_{\rm b,vir}$. $M_{\rm vir}$ can be estimated using the equation for the virial speed:

$$v_{\rm vir} = [GM_{\rm vir}H(z)]^{1/3}\left[\frac{\Delta_{\rm c}(z)}{2}\right]^{1/6}, \tag{1}$$

where $H(z)$ is the Hubble parameter and $\Delta_{\rm c}(z)$ is the critical overdensity for virialisation. You can assume that the virial speed is $v_{\rm vir} = v_{\rm rot}$ and take $H(z = 4) = 423.7\,{\rm km\,s^{-1}\,Mpc^{-1}}$, valid for a Planck 2018 cosmology. Paying attention to use the appropriate value for $\Delta_{\rm c}(z)$, calculate $M_{\rm vir}$ and $f_{\rm b,R_{vir}}$. For the latter, assume that the baryonic mass of the disc does not grow significantly beyond $R_{\rm out}$.

(iii) By comparing the baryon fraction obtained in (ii) with the cosmological baryon fraction, $f_{\rm b} = 0.16$, estimate the maximum mass of gas that could be hosted in the circumgalactic medium (CGM) of this galaxy. Do you expect this gas to be roughly at the virial temperature? Motivate your answer.

E2

Consider the progenitor of a supermassive black hole (SMBH), formed at redshift $z = 20$ with a mass $M_{\rm BH,0} = 1 \times 10^3\,M_\odot$. The SMBH is surrounded by a medium from which it can accrete gas at a high rate. As is customary, we assume that the accretion rate $\dot{M}_{\rm acc}$ equals the black hole's growth rate $\dot{M}_{\rm BH}$ and that the luminosity of the SMBH is $\dot{E} = \epsilon_{\rm rad}\dot{M}_{\rm acc}c^2$, with the radiative efficiency $\epsilon_{\rm rad} = 0.1$.

(i) Assuming that the black hole maintains an average Eddington ratio $f_{\rm Edd} = \dot{M}_{\rm acc}/\dot{M}_{\rm Edd} = 0.3$, where $\dot{M}_{\rm Edd} = L_{\rm Edd}/(\epsilon_{\rm rad}c^2)$ and $L_{\rm Edd} = \frac{4\pi c G m_{\rm p}}{\sigma_{\rm T}} M_{\rm BH}$, show that the black hole mass grows as

$$M_{\rm BH} = M_{\rm BH,0}\exp\left(t/\tau_{\rm s}\right) \tag{2}$$

with

$$\tau_{\rm s} = \frac{\epsilon_{\rm rad} c \sigma_{\rm T}}{4\pi G m_{\rm p} f_{\rm Edd}}. \tag{3}$$

Calculate the value of $\tau_{\rm s}$ and the value of the *cosmic* time, $t_{\rm f}$, at which the SMBH has reached a mass of $M_{\rm f} = 5 \times 10^8\, M_\odot$.
For the Thomson cross section, you can use $\sigma_{\rm T} = 6.65 \times 10^{-25}\,{\rm cm}^2$. The cosmic time corresponding to $z = 20$ is $t_{\rm i} = 0.18\,{\rm Gyr}$.

(ii) By assuming that the accretion of the SMBH essentially stops at the cosmic time $t_{\rm f}$, calculate the radiative energy released by the black hole during its entire growth period.

(iii) Then, calculate the energy released by all the Type-II supernovae that exploded in the galaxy that hosts the SMBH if the total stellar mass at $t_{\rm f}$ is $M_{\rm f} = M_*(t_{\rm f}) = 2 \times 10^{11}\, M_\odot$.
For this latter calculation, assume that the supernova rate is

$${\rm SNR}(t) = 0.01 \left[\frac{{\rm SFR(t)}}{M_\odot {\rm yr}^{-1}}\right] {\rm yr}^{-1}, \tag{4}$$

where SFR(t) is the star formation rate at cosmic time t and that $E_{\rm SN} = 1 \times 10^{51}$ erg is the standard energy release of a single supernova. Also assume instantaneous recycling approximation (IRA) with a return fraction $\mathcal{R} = 0.2$. Discuss how the energy output from supernovae compares with the energy output from the SMBH.
Tip: To perform this calculation, you can use the equation for the evolution of the stellar mass, $\frac{{\rm d}M_*}{{\rm d}t} = {\rm SFR} - \dot{\rm M}_{\rm ret}$, with $\dot{M}_{\rm ret}$ the return rate, to be modified given the IRA assumption, and integrate it in time. Note that the knowledge of the shape of the SFR or the SNR as a function of time is not needed for this calculation as you only need to the total number of supernovae exploded.

(iv) We finally estimate the redshift $z_{\rm f}$ corresponding to $t_{\rm f}$. In order to do this, we need an analytical function of the cosmic time as a function of z, i.e. $t(z)$. A good approximation for a ΛCDM universe at $z > 1$ can be found by integrating the first Friedmann equation:

$$\dot{a}^2 = \frac{8\pi G}{3}\rho a^2, \tag{5}$$

where a and $\dot{a}$ are the scale factor and its time derivative, respectively, and considering $\rho = \rho_{\rm m}$, i.e. only the contribution of the matter density and neglecting that of the dark energy. A useful procedure is to first use the relation $\rho_{\rm m} = \rho_{\rm m,0} a^{-3}$ to integrate the Friedman equation and then substitute the relation between a and z.
Tip: Remember also that $\rho_{\rm m,0} = \rho_{\rm crit,0}\Omega_{\rm m,0}$, with $\rho_{\rm crit,0} = \frac{3H_0^2}{8\pi G}$, where H_0 is the Hubble constant that you can assume to be $70\,{\rm km\,s^{-1}\,Mpc^{-1}}$, and $\Omega_{\rm m,0} = 0.315$.

Physical constants

Constants	Symbol	Value	Units (cgs)
Speed of light	c	$2.99792458 \times 10^{10}$	$\mathrm{cm\,s^{-1}}$
Gravitational constant	G	6.67408×10^{-8}	$\mathrm{cm^3\,g^{-1}\,s^{-2}}$
Boltzmann constant	k_B	$1.38064852 \times 10^{-16}$	$\mathrm{erg\,K^{-1}}$
Planck constant	h	$6.626070040 \times 10^{-27}$	$\mathrm{erg\,s}$
Mass of electron	m_e	$9.10938356 \times 10^{-28}$	g
Mass of proton	m_p	$1.672621898 \times 10^{-24}$	g

Astrophysical standard values

Constants	Symbol	Value	Units
Parsec	pc	3.0856776×10^{18}	cm
Solar mass	$M_\odot$	1.9891×10^{33}	g
Sideral year	yr	3.155815×10^{7}	s
Thompson cross section	σ_T	6.65×10^{-25}	$\mathrm{cm^2}$